Further Pure Core – Partial Fractions

In single maths you learnt how to split two types of expressions into partial fractions; namely:

$$\frac{px+q}{(ax+b)(cx+d)} \equiv \frac{A}{ax+b} + \frac{B}{cx+d},$$

$$\text{and } \frac{px^2+qx+r}{(ax+b)(cx+d)^2} \equiv \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}.$$

In further maths we add

$$\frac{px^2+qx+r}{(ax+b)(cx^2+d)} \equiv \frac{A}{ax+b} + \frac{Bx+C}{cx^2+d}.$$

As before this becomes important in integrating certain rational functions in the form $\frac{p(x)}{q(x)}$.

1. Split the following into partial fractions:

   (a) $\dfrac{x+1}{(x^2+1)(x-1)}$. $\boxed{\frac{1}{x-1} - \frac{x}{x^2+1}}$

   (b) $\dfrac{3x+4}{(x^2+4)(x+2)}$. $\boxed{\frac{x+10}{4(x^2+4)} - \frac{1}{4(x+2)}}$

   (c) $\dfrac{1-3x}{(1+2x)(1+x^2)}$. $\boxed{\frac{2}{2x+1} - \frac{x+1}{x^2+1}}$

   (d) $\dfrac{16+2x+15x^2}{(1+x^2)(2-x)}$. $\boxed{\frac{x}{x^2+1} - \frac{16}{x-2}}$

   (e) $\dfrac{3x^2+4x-28}{(2x-1)(x^2+25)}$. $\boxed{\frac{2x+3}{x^2+25} - \frac{1}{2x-1}}$

   (f) $\dfrac{2x^2}{(x+2)(x^2+4)}$. $\boxed{\frac{x-2}{x^2+4} + \frac{1}{x+2}}$

2. Find the values of the following integrals:

   (a) $\displaystyle\int_1^2 \frac{2+3x^2}{x(x^2+2)}\,dx$. $\boxed{2\ln 2}$

   (b) $\displaystyle\int_0^1 \frac{1-2x-x^2}{(1+x)(1+x^2)}\,dx$. $\boxed{0}$

   (c) $\displaystyle\int_2^3 \frac{8+3x-x^2}{(x-1)(x^2+4)}\,dx$. $\boxed{\frac{13}{2}\ln 2 - \frac{3}{2}\ln 13}$